Different Level Solutions for Mango Problem

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The Problem One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a bowl full of mangoes.

1. Being hungry, he took 1/6 of the mangoes.
2. Later that same night, the Queen was hungry and couldn't sleep. She, too, found the mangoes and took 1/5 of what the King had left.
3. Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes.
4. Even later, his brother, the second Prince, ate 1/3 of what was then left.
5. Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

   How many mangos originally were in the bowl before the king took his first share of mangos?

Solution 1 (suitable for most elementary students)

In this solution we will use a backwards approach, and at hte same time a pictorial description available to most elementary students who understand fractions.

Let **M** represent a mango.

A- Since at the very end there were only three mangos **M M M** left for the servants, and since the Third Prince ate half of what was left from the Second Prince, the number of mangos left by Second Prince should be represented by **M M M M M M**.

B- Since the Second Prince had a third of mangos left from the First Prince it means that **M M M M M M** represents two thirds of mangos left by the First Prince, therefore the First Prince must have left as many as

M M M M M M M M M. (1)

mangos, which is 9.

C- Next, since the First Prince had $\frac{1}{4}$ of the mangos left by the Queen, the number of mangos in (1) must also represent $\frac{3}{4}$ of mangos left by the Queen, which means the Queen must have left as many as

M M M M M M M M M M M M. (2)

mangos, which is 12.

D- Since Queen had $\frac{1}{5}$ of the mangos left by the King, above (2) must also represent $\frac{4}{5}$ of the mangos left by the king, and hence the King must have left as many as

M M M M M M M M M M M M M M M. (3)

mangos, which is 15.

E- And finally, since the King had $\frac{1}{6}$ of the original number of mangos in the bowl, the above (3) also represents $\frac{5}{6}$ of the total number of mangoes originally were in the bowl .Therefore the original number of mangos in the bowl must be represented by

M M M M M M M M M M M M M M M M M M,

which is 18, and the solution of the problem

Solution 2(a) (perhaps for garde 7-8's):

In this solution we work forwards, and assume the target student knows that if N is a number, then $\frac{m}{n}$ portion of N is simply $\frac{m}{n} \times N$.

Let N represent the number of mangos originally in the bowl. Since the King had $\frac{1}{6}$ of mangos, he had left as many as $N - \frac{1}{6} N = \frac{5}{6} N$ mangos in the bowl. Since the Queen had $\frac{1}{5}$ of what King had left, she ate $\frac{1}{5}\left(\frac{5}{6}\right) N = \frac{1}{6} N$ mangoes and left as many as $\frac{5}{6} N - \frac{1}{6} N = \frac{2}{3} N$ for the First Prince. Since the Firs Prince had $\frac{1}{4}$ of mangoes left by the queen, he must have left $\frac{2}{3} N - \frac{1}{4}\left(\frac{2}{3}\right) N = \frac{1}{2} N$ mangos in the bowl. And likewise the Second Prince had $\frac{1}{3}$ of the remaining $\frac{1}{2} N$ mangos, so he left $\frac{1}{2} N - \frac{1}{3}\left(\frac{1}{2} N\right) = \frac{1}{3} N$ mangos. And finally the Third Prince had $\frac{1}{2}$ of these $\frac{1}{3} N$ mangoes, so he left as many as left $\frac{1}{3} N - \frac{1}{2}\left(\frac{1}{3}\right) N = \frac{1}{6} N = 3$ of mangoes for the servants. And $\frac{1}{6} N = 3$ implies $N = 18$. Hence there were 18 mangos in the bowl.

Solution 2(b) (suitable for garde 9's, or even 9 honors):

This solution is simply the translation of Solution 2(a) into an equation as follows:

Again let N represent the number of mangos originally in the bowl .Since the King had $\frac{1}{6}N$ mangoes he left $\frac{5}{6}N$ mangos in the bowl, therefore we have the following equation in terms of N

$$\frac{1}{2}\left\{\left[\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)-\frac{1}{4}\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)\right]-\frac{1}{3}\left[\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)-\frac{1}{4}\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)\right]\right\}=3$$

$$\left[\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)-\frac{1}{4}\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)\right]-\frac{1}{3}\left[\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)-\frac{1}{4}\left(\frac{5}{6}N-\frac{1}{5}\times\frac{5}{6}N\right)\right]=6$$

$$\left[\left(\frac{2}{3}N\right)-\frac{1}{4}\left(\frac{2}{3}N\right)\right]-\frac{1}{3}\left[\left(\frac{2}{3}N\right)-\frac{1}{4}\left(\frac{2}{3}N\right)\right]=6$$

$$\left[\frac{1}{2}N\right]-\frac{1}{3}\left[\frac{1}{2}N\right]=6$$

$$\frac{1}{3}N=6\rightarrow N=18$$

Solution 3 (for grades 7-8's):

In this solution we again work backwards, but we assume student knows if a number N is $\frac{m}{n}$ portion of a total number T, then $T=N\div\frac{m}{n}=N\times\frac{n}{m}$.

Let K, Q , $P1$, $P2$, and $P3$ resent the number of mangos in the bowl before the before King, the Queen, First Prince, Second Prince, and the Third Prince ate their share of mangos, respectively. Since the servants were left with 3 mangoes and First Prince had half of the mangos in the bowl, it follows that $P3=6$. Since the Second Prince had $\frac{1}{3}$ of what was left for him it follows that $P2=6\div\frac{2}{3}$ =9. Similarly, $P1=9\div\frac{3}{4}12$, $Q=12\div\frac{4}{5}$ =15, and $K=15\div\frac{5}{6}=18$. Therefore the solution is 18.

Solution 4:

This solution isn't my work, but from a video on line and uses the method of educated guess and check. But I have changed the wording for better understanding.

Since the King ate $\frac{1}{6}$ of the mangos originally in the bowl, it is safe to assume that the original number of the mangos has been a multiple of 6. Since the most obvious such number is 6 itself, in which case the King had just one mango and left 5of them. Then the Queen had $\frac{1}{5}$ of 5, which means just one and left 4 in the bowl. Then the first Prince had $\frac{1}{4}$ of the 4, meaning another one and left with 3 in the bowl. After that the Second Prince had $\frac{1}{3}$ of 3 mangos which means again 2, leaving 2. Then the Third Prince took half of 2, meaning 1 and left 1 for the servants. Since we know that in actual fact there were 3 mangoes left for servants, so in order to see how many mangos each member of Royal family has had, we should multiply everyone's share of mango in our educated guess by 3. It therefore follows that every member of the royal family has had 3 mangos. Considering that the servants also had 3, the answer to the problem must be $3 + 5(3) = 18$.