

The Exact Values Of Trig Ratios For Special Angles

The following is how students of PMath 11 and PMath 12 can set up a table for the exact values of special angles $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi$, in about a minute.

Step 1: Line up the above special angles over a number line, and below each of the angles record the fraction $\frac{\sqrt{\quad}}{2}$ (yes, with an empty radical), and with $\sin \theta$ on the far left below θ ,

| | | | | | | | | | |
|---------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| θ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| -----> | | | | | | | | | |
| $\sin \theta$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ | $\frac{\sqrt{\quad}}{2}$ |

Step 2: Start counting up from zero to 4, and then down to 0 and fill the radicals with these counting numbers in the same order,

| | | | | | | | | | |
|---------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| θ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| -----> | | | | | | | | | |
| $\sin \theta$ | $\frac{\sqrt{0}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{4}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{0}}{2}$ |

Step 3: Simplify the second row, and record the result as a third row,

| | | | | | | | | | |
|---------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| θ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| -----> | | | | | | | | | |
| $\sin \theta$ | $\frac{\sqrt{0}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{4}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{0}}{2}$ |
| ----- | | | | | | | | | |
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |

Step 4: In order to write the values of $\cos \theta$ as a fourth row proceed as follows:
 As the first five entries of this row, enter the first five entries of the third row,
but in reverse order. Then, as the remaining four values of $\cos \theta$, record the opposite
 values of the first four of five numbers you recorded earlier, in such a way that the entries
 of the fourth row show up in descending order.

| | | | | | | | | | |
|---------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|----------------------|
| θ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| -----> | | | | | | | | | |
| $\sin \theta$ | $\frac{\sqrt{0}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{4}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{0}}{2}$ |
| ----- | | | | | | | | | |
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| ----- | | | | | | | | | |
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 |

Step 5: In order to write the values of $\tan \theta$ divide the corresponding values of the third
 row by the fourth row,

| | | | | | | | | | |
|---------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|----------------------|
| θ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| -----> | | | | | | | | | |
| $\sin \theta$ | $\frac{\sqrt{0}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{4}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{0}}{2}$ |
| ----- | | | | | | | | | |
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| ----- | | | | | | | | | |
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 |
| ----- | | | | | | | | | |
| $\tan \theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ | $-\sqrt{3}$ | -1 | $-\frac{1}{\sqrt{3}}$ | 0 |

Remark One can add three rows for ratios $\cot \theta$, $\sec \theta$, and $\csc \theta$, by recording the
 reciprocal of the values in rows four, three, and the fifth row, respectively.