An Infinite Sequence of Twin Composite Numbers

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According to Underwood Dudly in his Number Theory Textbook, in 1509 DeBouvelles claimed that for all natural numbers n one or both of the twin numbers $6n-1$ and $6n+1$ are prime numbers. Here by twin numbers we mean consecutive odd integers. Dudly then leaves the following exercise for verification:

Exercise

(a) Show that DeBouvelles was wrong.

(b) Moreover, show that there are infinitely many numbers n for which both $6n-1$ and $6n+1$ are composite numbers

Since the superior part (b) of the Exercise is a reminiscent of the still popular and ancient open "Twin Prime" problem, I became interested to verify part (b), by figuring out the following twin composite sequence,

Let k be any positive integer, and set $n = 36(5k+2)^2$. Then for each such n both consecutive (twin) numbers

$6n-1 = (30k+11)(30k+13)$ and $6n+1 = 5(180k^2+144k+29)$

are composite . Moreover, if you wish the second integer $6n+1$ to look a little more composite than above choose $k = 29j$ for $j = 1, 2, \ldots$, to get

$6n-1 = (870j+11)(870j+13)$ and

$6n+1 = 5(29)(151380j^2+4176j+1)$.

You could even make the first integer to look more composite by setting $j = 11i$ or $= 13i$, $i = 1, 2, \ldots$. Then your pair of sequence of composite twin numbers

(A) $6n-1 = 11(9570i+1)(9570i+13)$
$6n+1 = 5(29)(151380j^2+4176j+1)$, or

(B) $6n-1 = 13(9570i+1)(9570i+1)$
$6n+1 = 5(29)(151380j^2+4176j+1)$.