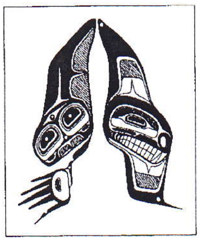

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A Comment on the Distance from a Point to a Line

A few months ago, a colleague teaching Principles of Math 11 asked me for a proof of the distance formula from a point to a line, and I made up the following for her.

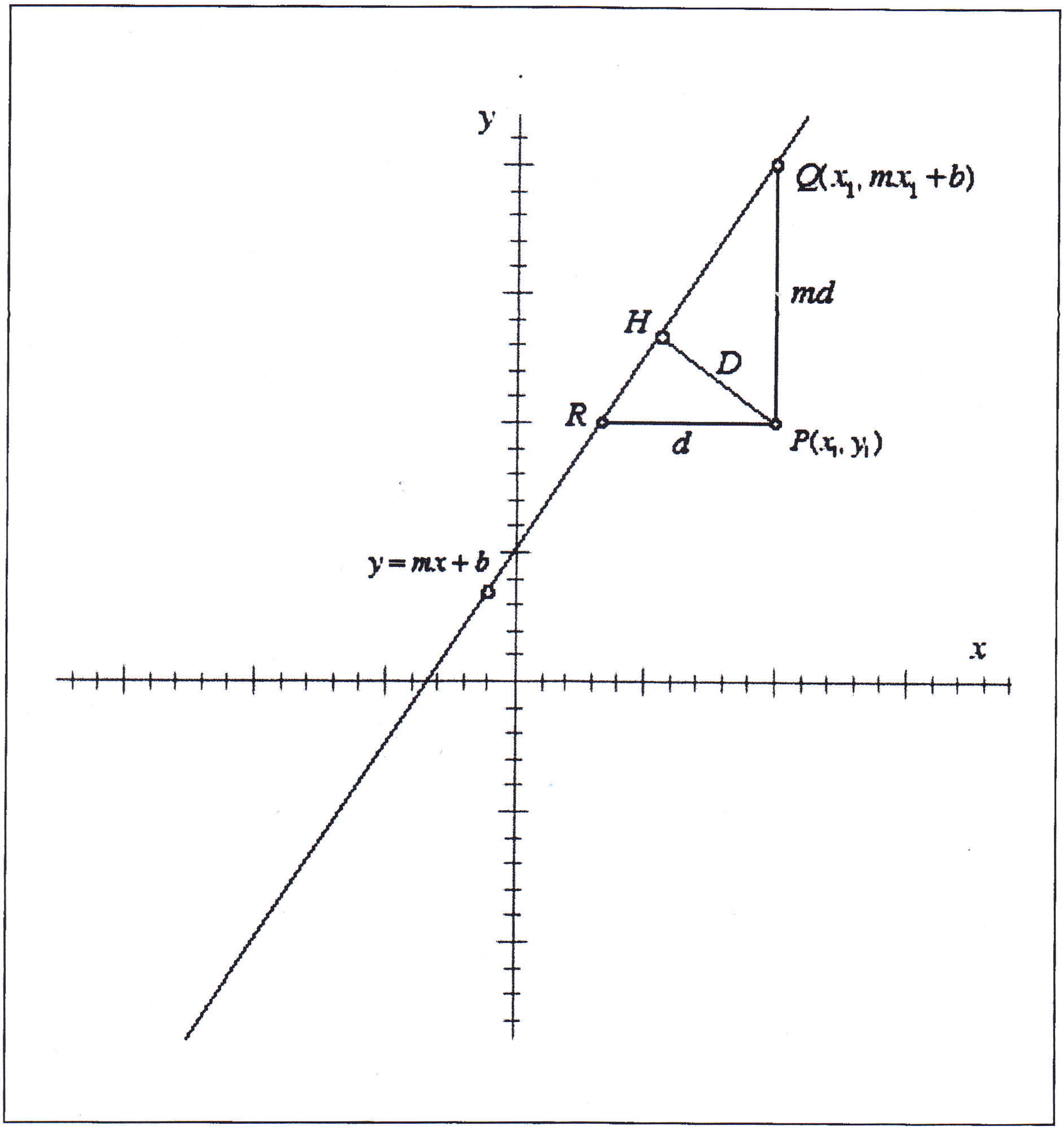
Compared to the proofs presented by John Assadi in the spring 2005 edition of *Vector*, this one doesn't use any trigonometry. Therefore it is easier and more accessible to interested PM 11 students who haven't yet learned about trigonometric identities.

Here, I use only the Pythagorean Theorem and similarity of triangles. Since it is easy to verify the formula for vertical or horizontal lines, I will assume the line is neither vertical nor horizontal.

We want to derive the formula for the distance from a point $P(x_1,y_1)$ to a line with equation L: $y = mx + b$, say the distance $D = PH$ as in the figure (on next page) First, draw vertical and horizontal lines from P so that they intersect the line L at Q and R respectively. If we set $RP = d$, then by definition of slope, we will have $PQ = |m|d$, and from the Pythagorean theorem in the triangle PQR, we will also have $QR = (\sqrt{1+m^2})d$.

Next, from similarity of the right triangles RPQ and RHP we can write $\frac{D}{|md|} = \frac{d}{(\sqrt{1+m^2})d}$ which implies $D = \frac{|md|}{\sqrt{1+m^2}}$. Because P and Q are on the same vertical line, we have $|md| = |y_1 - mx_1 - b|$, and the formula $D = \frac{|y_1 - mx_1 - b|}{\sqrt{1+m^2}}$ is obtained.

Given the equation of a line is given in the form $Ax + By + C = 0$, we substitute $m = -\frac{A}{B}$ and $b = -\frac{C}{B}$ for D, and obtain the more standard form $D = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.


y
x
Q(x1, mx1 +b)
md
H
D
R
d
P(x1, y1)
y = mx + b