Factoring Trinomials, and quadrinomials

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The objective of this short note is to show a method to factor trinomials and quadriomials with a non-factorable leading coefficient $a > 1$, that I figured for my students back in late 90's, and also posted it on the bcamt email list serve for Secondary teachers around then. Here is how it works,

Example 1: To factor the trinomial $2x^2 - 3x - 14$, we can write,

$$2x^2 - 3x - 14 = \frac{4x^2-6x-28}{2} = \frac{[(2x)^2-3(2x)-28]}{2} = \frac{[(2x)-7][(2x)+4]}{2} = \frac{2[2x-7][x+2]}{2} = (2x-7)(x+2) .$$

Here is a little more interesting one,

Example 2: To factor the trinomial $6x^2 + x - 2$, we will write,

$$6x^2 + x - 2 = \frac{36x^2+6x-12}{6} = \frac{[(6x)^2+(6x)-12]}{6} = \frac{[(6x)+4][(6x)-3]}{6} = \frac{6[3x+2][2x-1]}{6} = (3x+2)(2x-1) .$$

And here is how you could factor a quadrinomial with a non-factorable leading coefficient $a > 1$,

Example 3:

$$\begin{aligned} &6x^3 + 5x^2 - 2x - 1 = 36\ (6x^3 + 5x^2 - 2x - 1)/36 = \\ &[(6x)^3 + 5(6x)^2 - 12(6x) - 36]/36 = \\ &(1/36)\ [A^3 + 5A^2 - 12A - 36], \quad \text{where } A = 6x. \end{aligned} \tag{1}$$

Since the leading coefficient of the polynomial $A^3 + 5A^2 - 12A - 36$ in the square bracket is 1, we can use the integral zero theorem (that used to be on BC secondary curriculum back in late 90's) and factor this polynomial in terms of A as follows

$$A^3 + 5A^2 - 12A - 36 = (A-3)(A+6)(A+2) .$$

Now substituting the right hand side of the above relation for the last square bracket in (1), and then substituting back $6x$ for A afterwards, we get the following required factorization of the original polynomial as,

$$6x^3 + 5x^2 - 2x - 1 = (1/36)[A^3 + 5A^2 - 12A - 36] = (1/36)[(A-3)(A+6)(A+2)]$$
$$= (1/36)(6x-3)(6x+6)(6x+2) = (2x-1)(x+1)(3x+1).$$

I close this note by pointing out that later in early 2000's I generalized the above method very formally (with graphical interpretations and proofs) to any integral polynomial of degree $n = 3, 4, \ldots$ with a non-factorable leading coefficient $a > 1$. I called the resulting method "A Multipliers Algorithm to Factor Polynomials", and the Algorithm can be visited as article # 9 of this same Algebra section of the website.

It should be clear to readers familiar with the "Rational Zero Theorem" and the "Integral Zero Theorem" that my above method makes the use of former theorem redundant altogether. That is, if you use this method you never need to use Rational Zero theorem to factor integral polynomials, or solve integral polynomial equations. Indeed article #9 of this same section is a living proof that a method I have called "Multiplier's Algorithm" works for generalized integral polynomials having rational solutions .